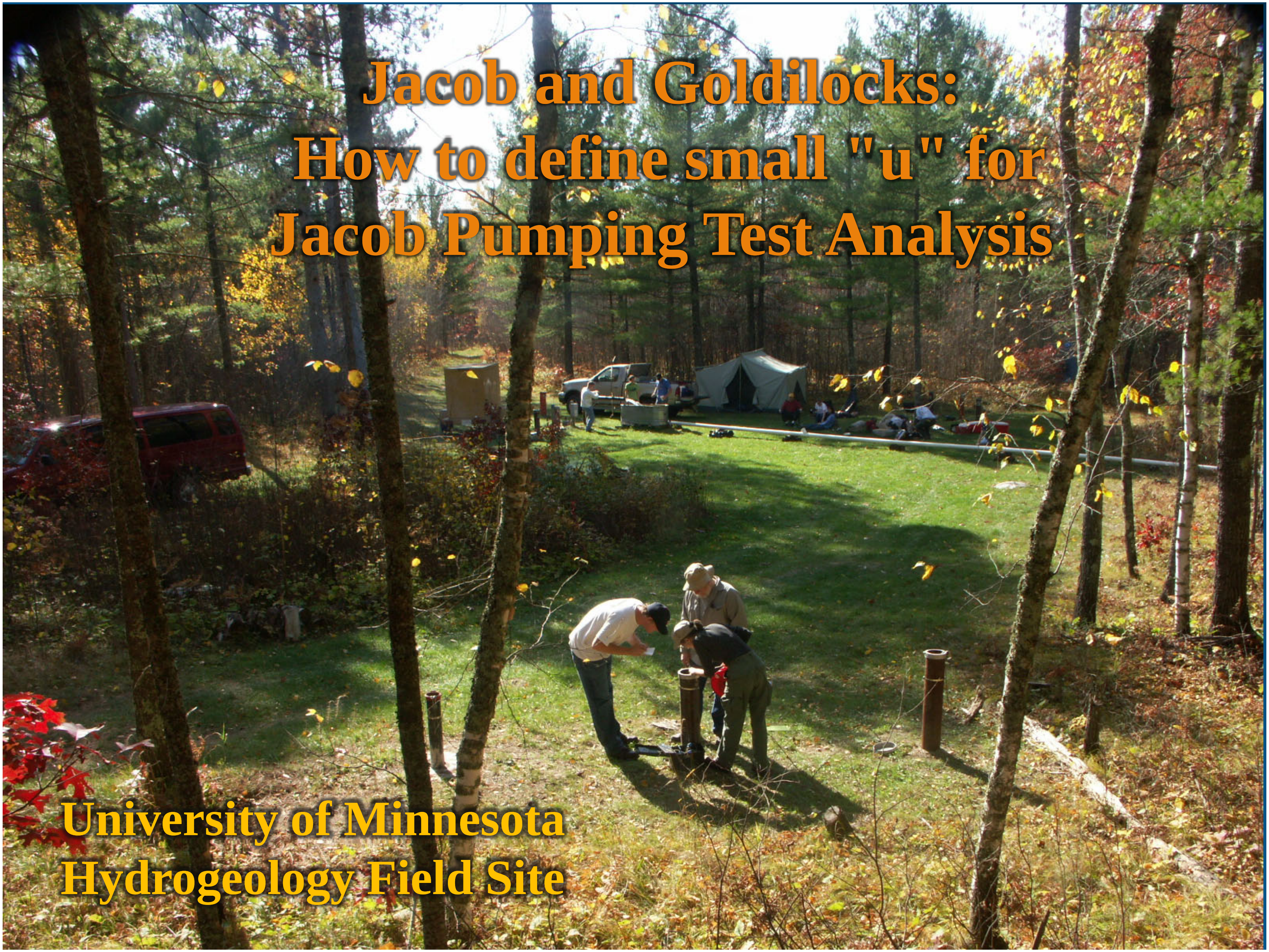
Jacob and Goldilocks: How to define small "u" for Jacob Pumping Test Analysis



University of Minnesota
Hydrogeology Field Site

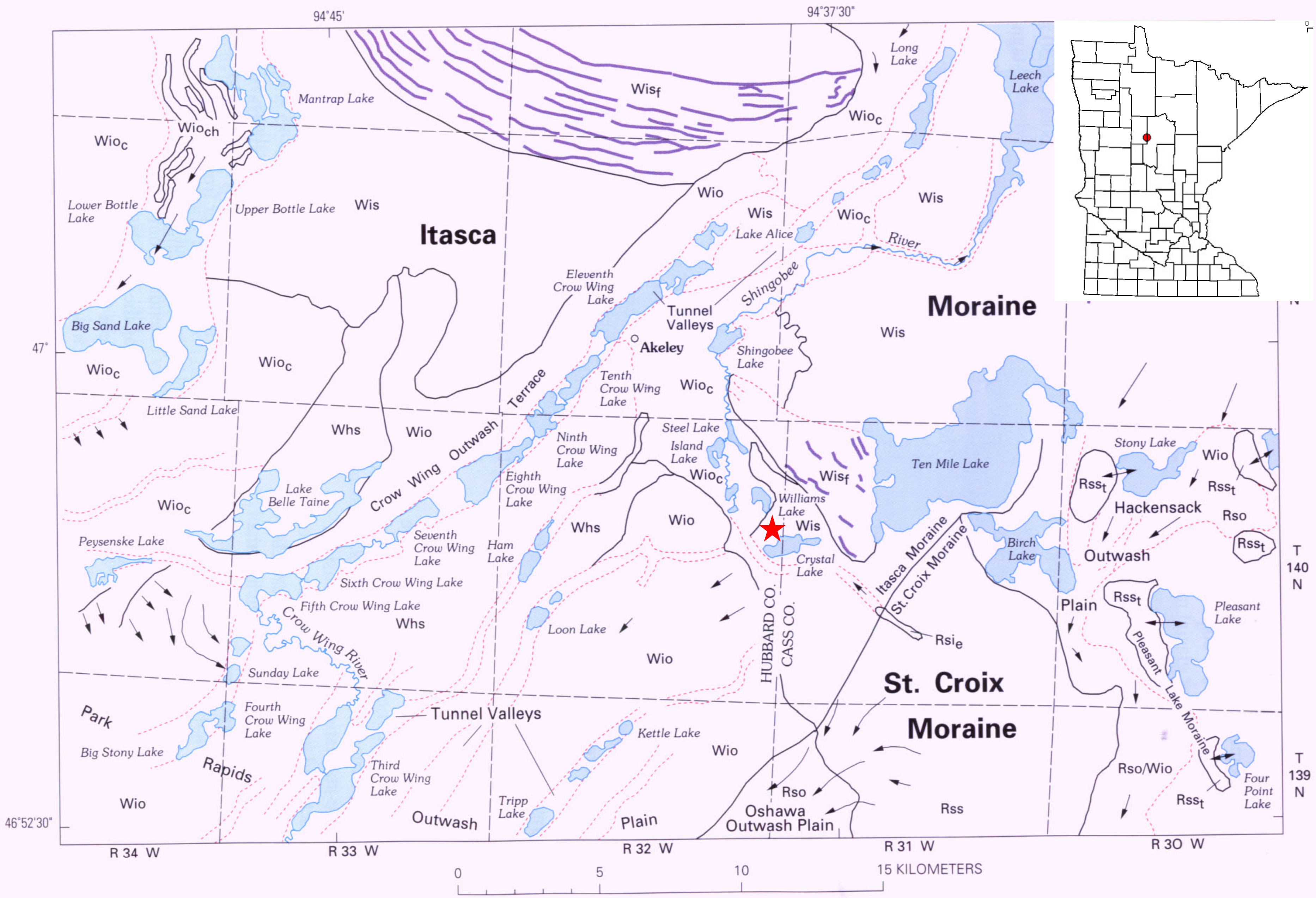
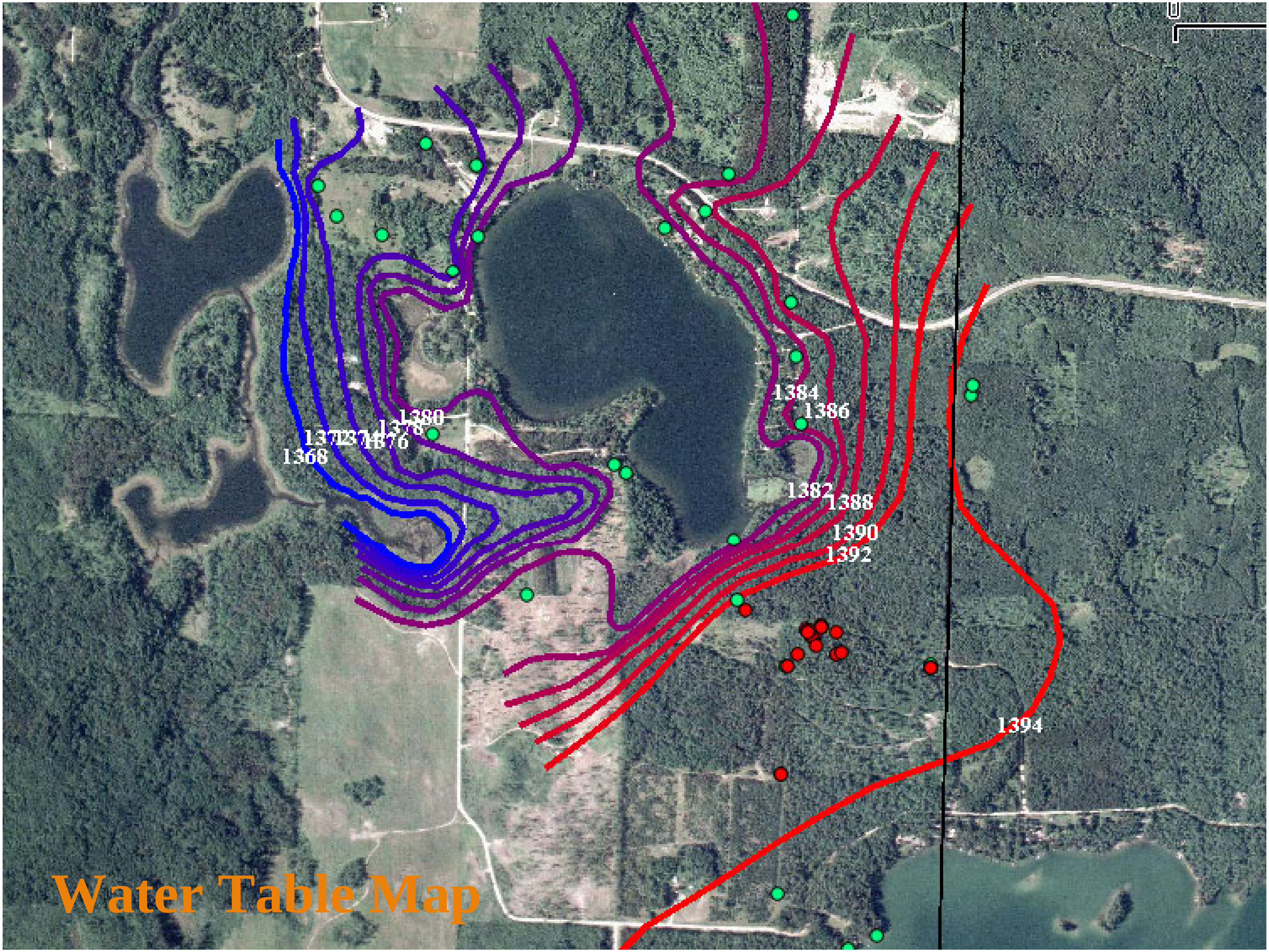
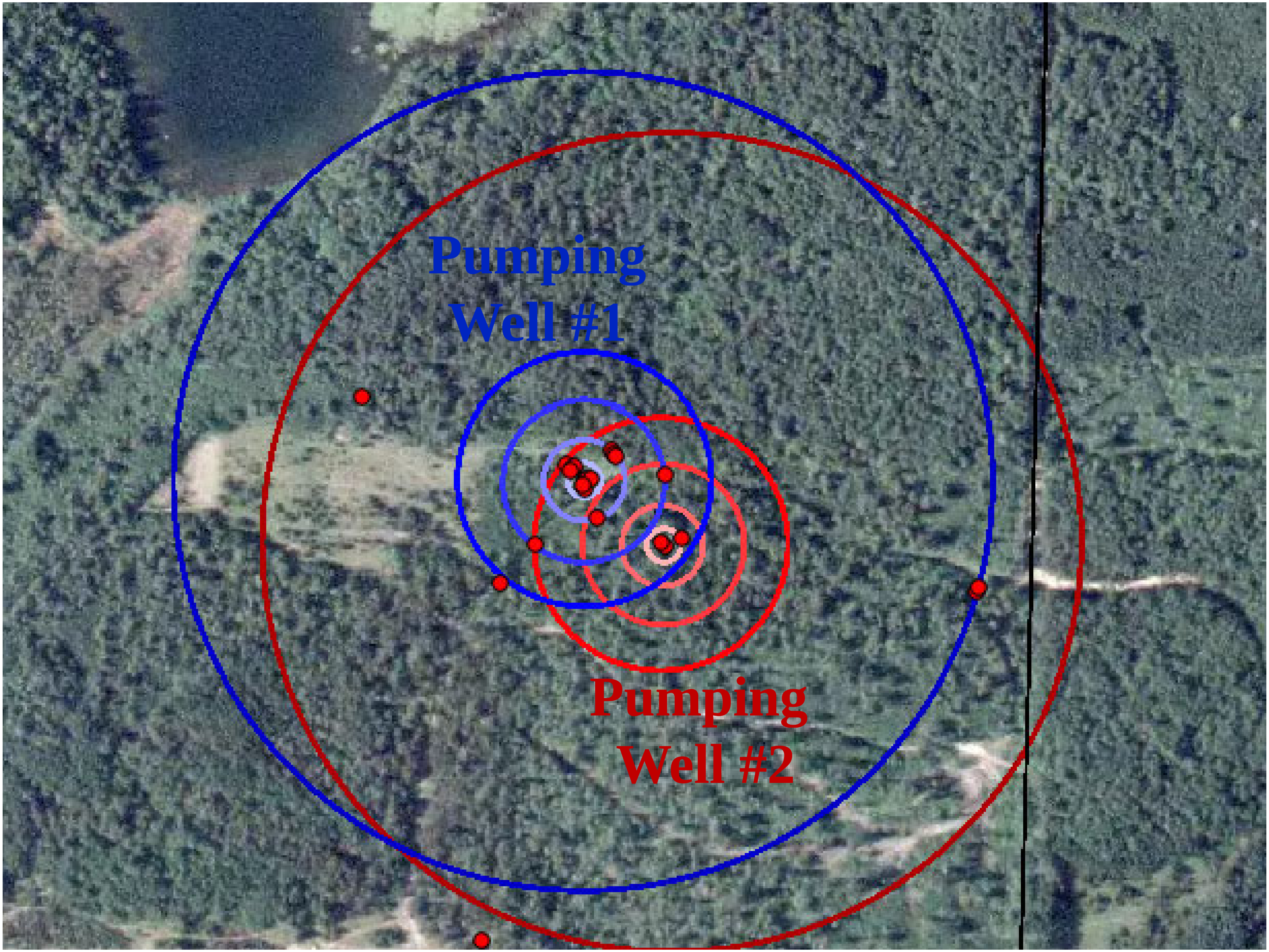
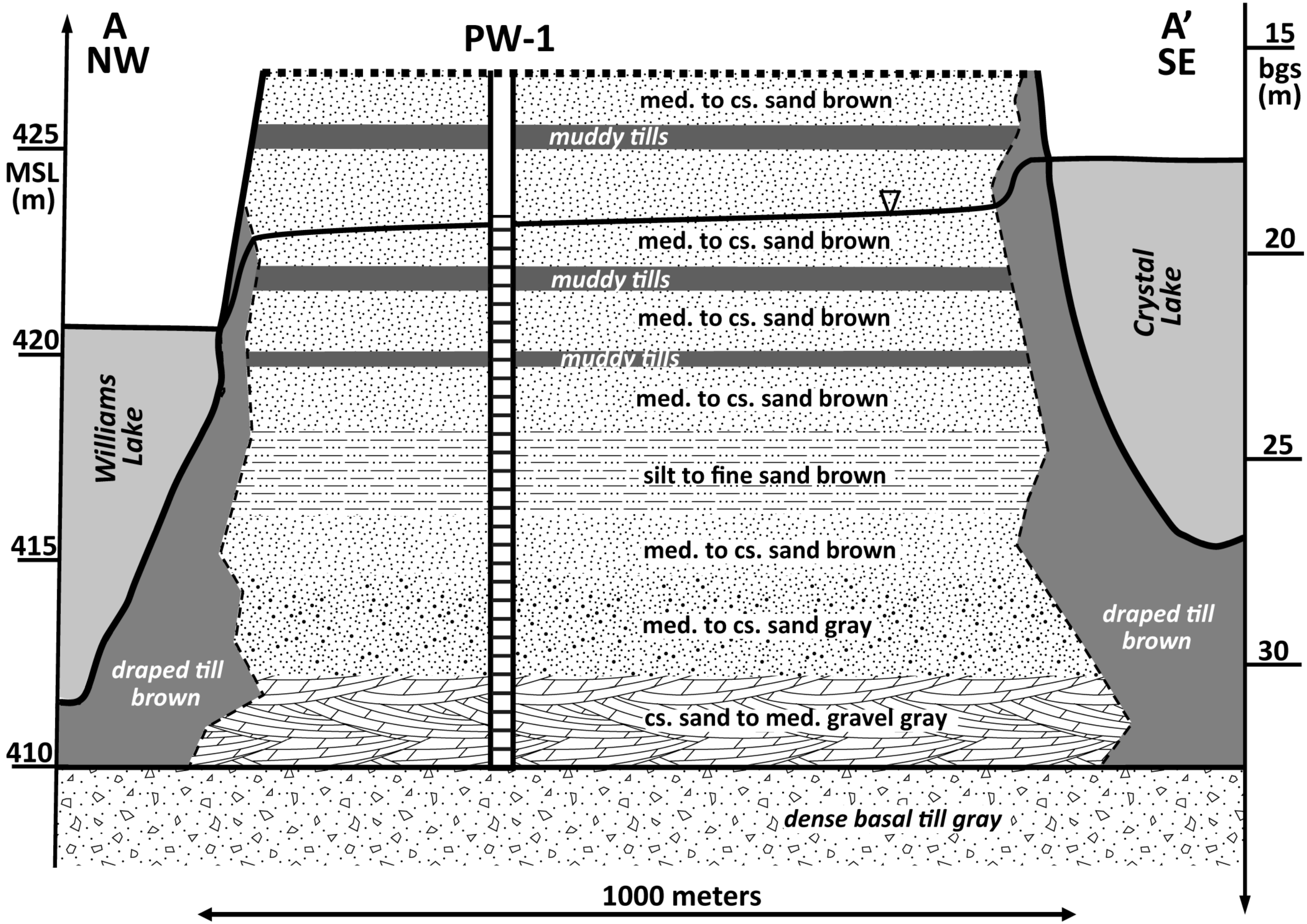


Figure 2. Surficial geologic map of the Itasca/St. Croix moraine interlobate area. Map area corresponds to inset on figure 1. (Compiled from Norton (1983), Mooers (1988), Mooers and others (1990), Wright (1993), and Mooers (unpub. data)).


Water Table Map
1368
1376
1378
1380
1382
1384
1386
1388
1390
1392
1394


Pumping Well #1
Pumping Well #2


A
NW
PW-1
A'
SE
15
bgs
(m)
425
MSL
(m)
420
415
410
20
25
30
med. to cs. sand brown
muddy tills
med. to cs. sand brown
muddy tills
med. to cs. sand brown
muddy tills
med. to cs. sand brown
silt to fine sand brown
med. to cs. sand brown
med. to cs. sand gray
cs. sand to med. gravel gray
dense basal till gray
Williams Lake
Crystal Lake
draped till brown
draped till brown
1000 meters

Cooper Jacob Analysis

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}}{u}\,du$$

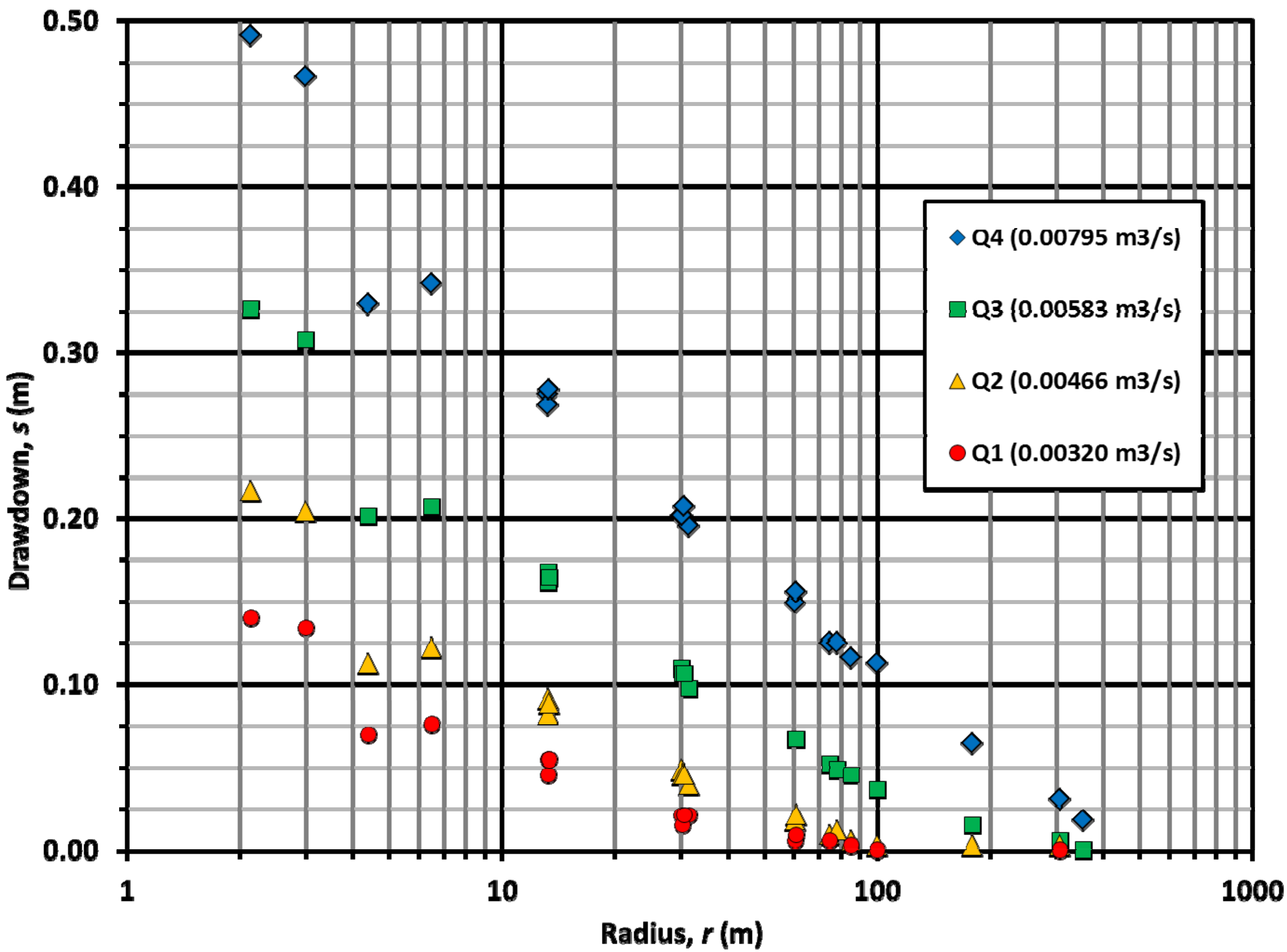
$$s = \frac{Q}{4\pi T}\left(-0.577216 - \ln(u) + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \frac{u^5}{5\cdot 5!}\cdots\right)$$

$$u = \frac{r^2 S}{4Tt}$$

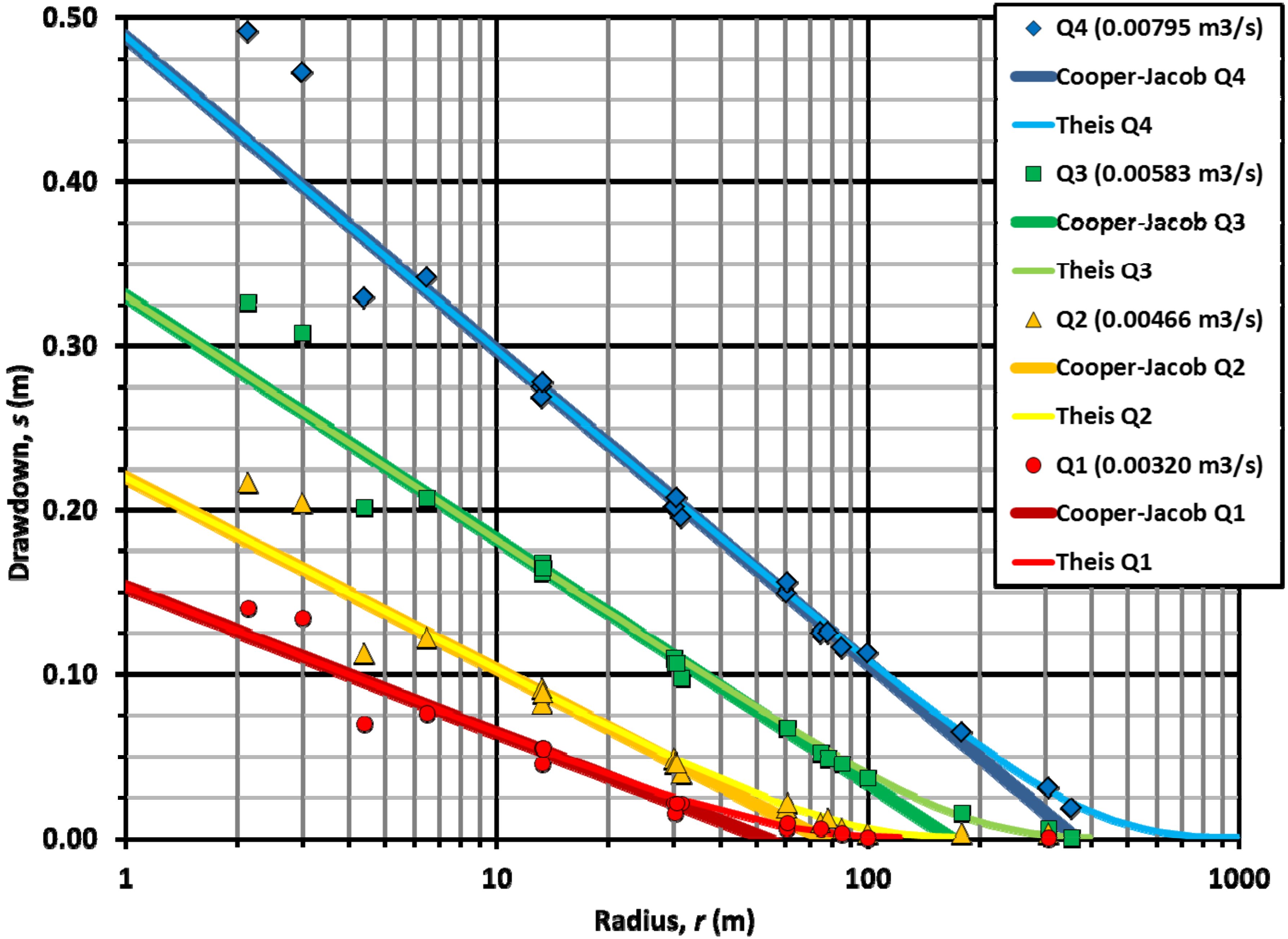
u **is small if**
radius *r* is large or time *t* is long

$$s = \frac{Q}{4\pi T}\left(-0.577216 - \ln(u)\right)$$

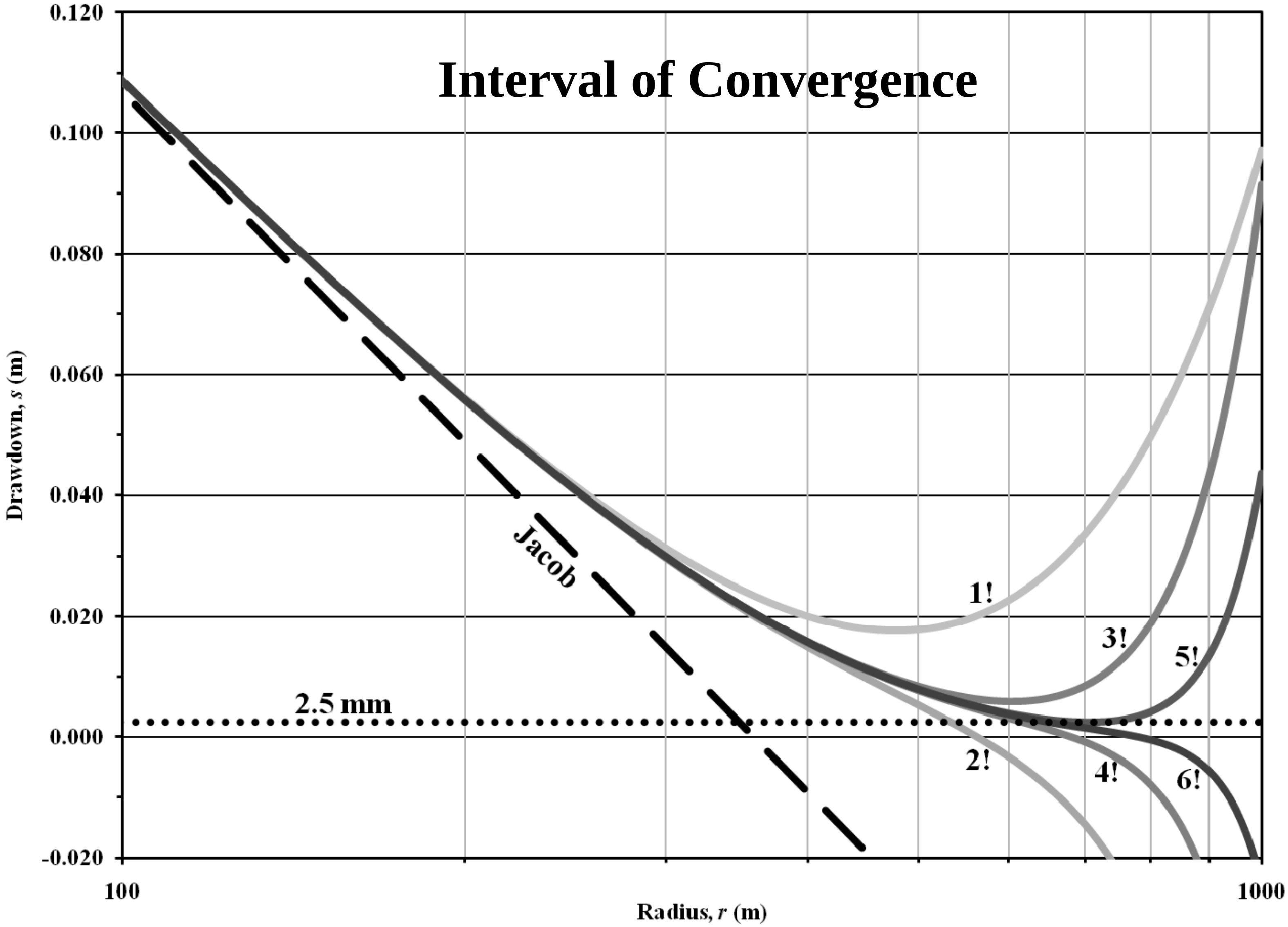
but how *small* is small enough?



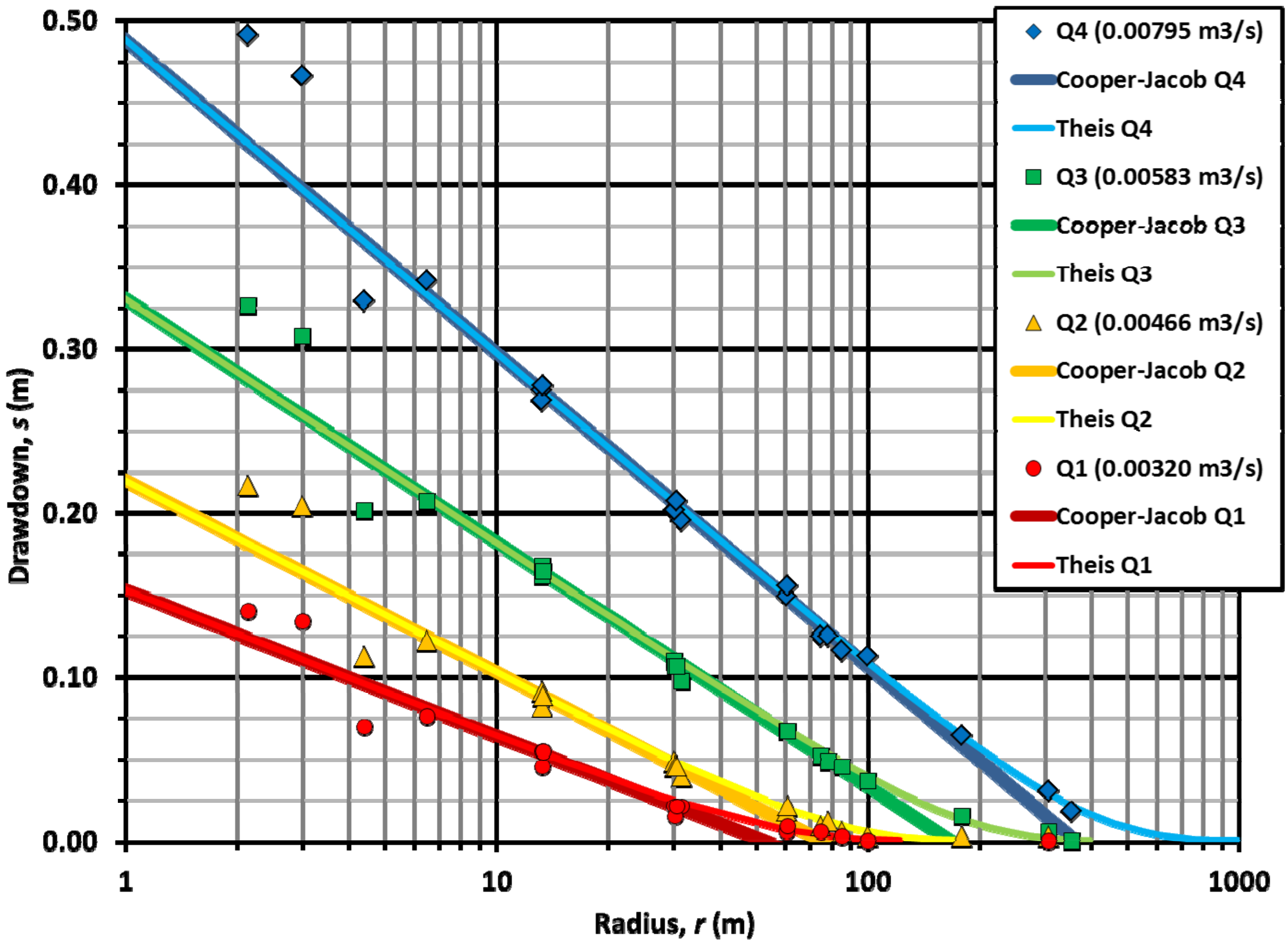
0.50
0.40
0.30
0.20
0.10
0.00
Drawdown, s (m)
1
10
100
1000
Radius, r (m)
Q4 (0.00795 m3/s)
Q3 (0.00583 m3/s)
Q2 (0.00466 m3/s)
Q1 (0.00320 m3/s)



0.50
0.40
0.30
0.20
0.10
0.00
Drawdown, s (m)
1
10
100
1000
Radius, r (m)
Q4 (0.00795 m3/s)
Cooper-Jacob Q4
Theis Q4
Q3 (0.00583 m3/s)
Cooper-Jacob Q3
Theis Q3
Q2 (0.00466 m3/s)
Cooper-Jacob Q2
Theis Q2
Q1 (0.00320 m3/s)
Cooper-Jacob Q1
Theis Q1



$$s = \frac{Q}{4\pi T}\left(-0.577216 - \ln(u) + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \frac{u^5}{5\cdot 5!}\cdots\right)$$



Q4 (0.00795 m3/s)
Cooper-Jacob Q4
Theis Q4
Q3 (0.00583 m3/s)
Cooper-Jacob Q3
Theis Q3
Q2 (0.00466 m3/s)
Cooper-Jacob Q2
Theis Q2
Q1 (0.00320 m3/s)
Cooper-Jacob Q1
Theis Q1
Drawdown, s (m)
0.50
0.40
0.30
0.20
0.10
0.00
1
10
100
1000
Radius, r (m)

Divergence of data at small radii

$$\mathrm{Re} = \frac{q\rho d_{10}}{\phi\mu}$$

turbulent flow starts at <0.5 m at this site

Impact of non-horizontal flow

Horizontal permeability is given by arithmetic mean:

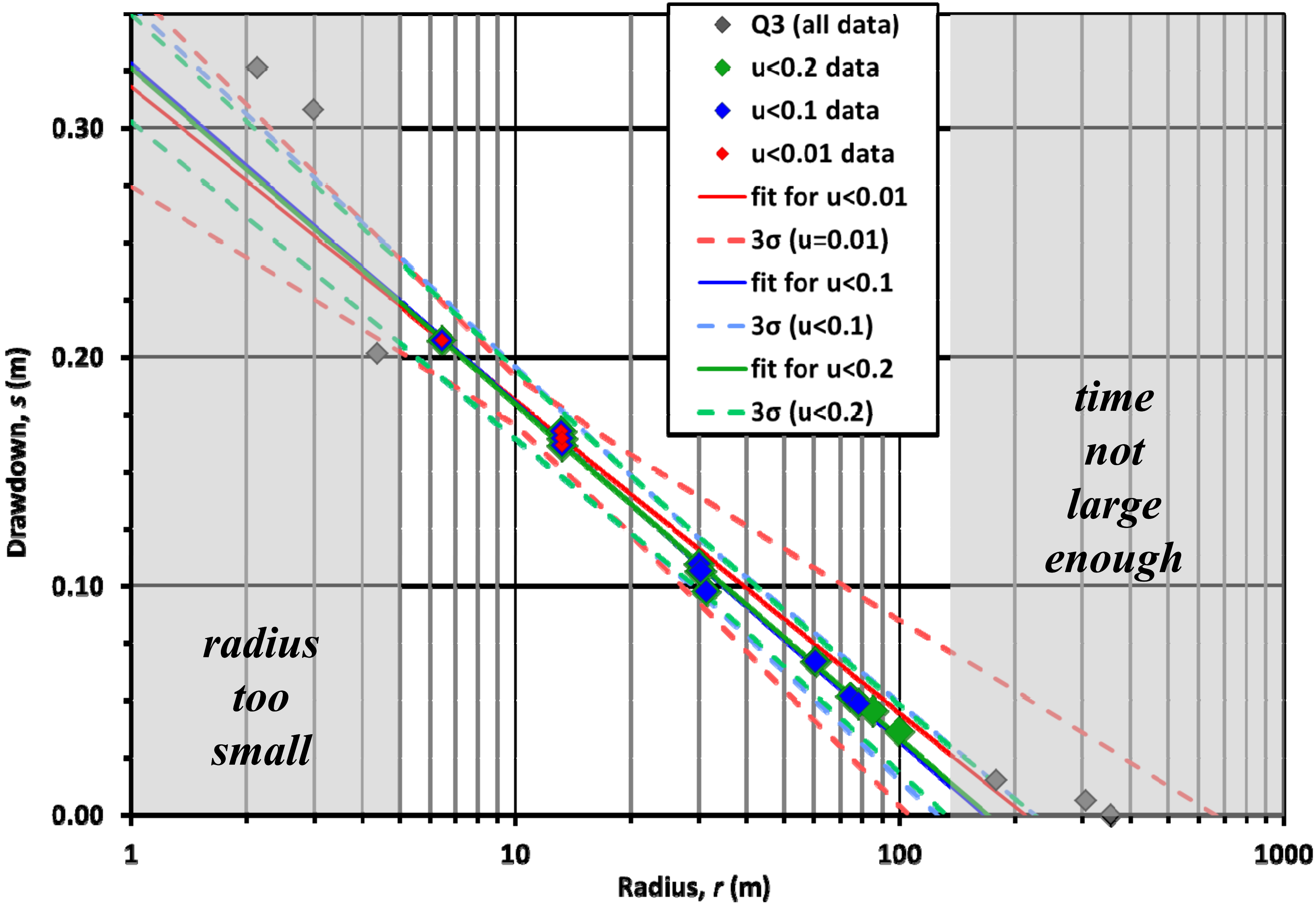
$$K_x = (\Sigma K_{xi}\, b_i)/b$$

K_x controlled by highest permeability layers

Vertical permeability is given by harmonic mean:

$$k_z = b/(\Sigma b_i/k_{zi})$$

K_z controlled by lowest permeability layers


Q3 (all data)
u<0.2 data
u<0.1 data
u<0.01 data
fit for u<0.01
3σ (u=0.01)
fit for u<0.1
3σ (u<0.1)
fit for u<0.2
3σ (u<0.2)
time not large enough
radius too small
Drawdown, s (m)
0.30
0.20
0.10
0.00
1
10
100
1000
Radius, r (m)

Do you know where our *water* comes from?

Hydro Camp

since 1995

University of Minnesota **Hydrogeologists** do.