Modeling transit time distributions in MODPATH

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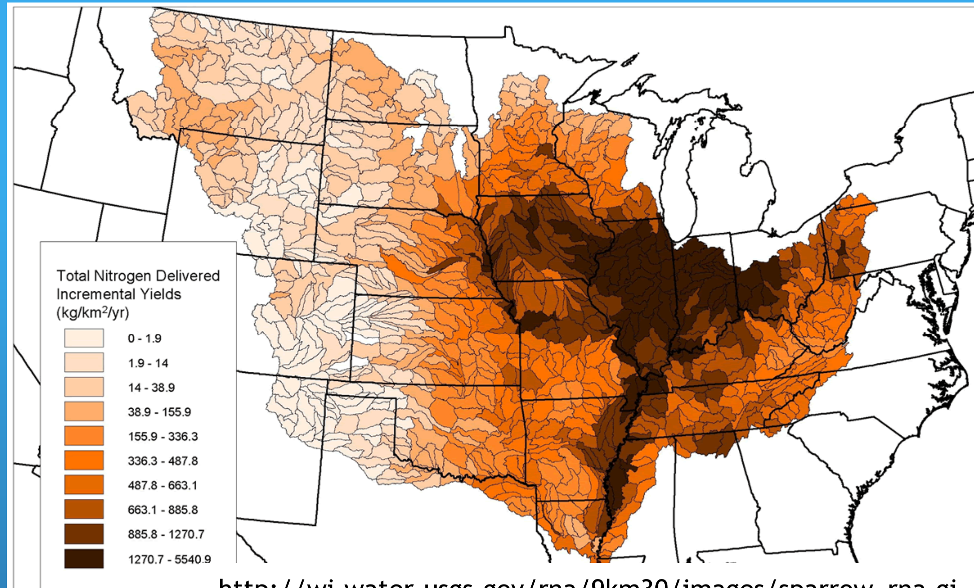
Outline

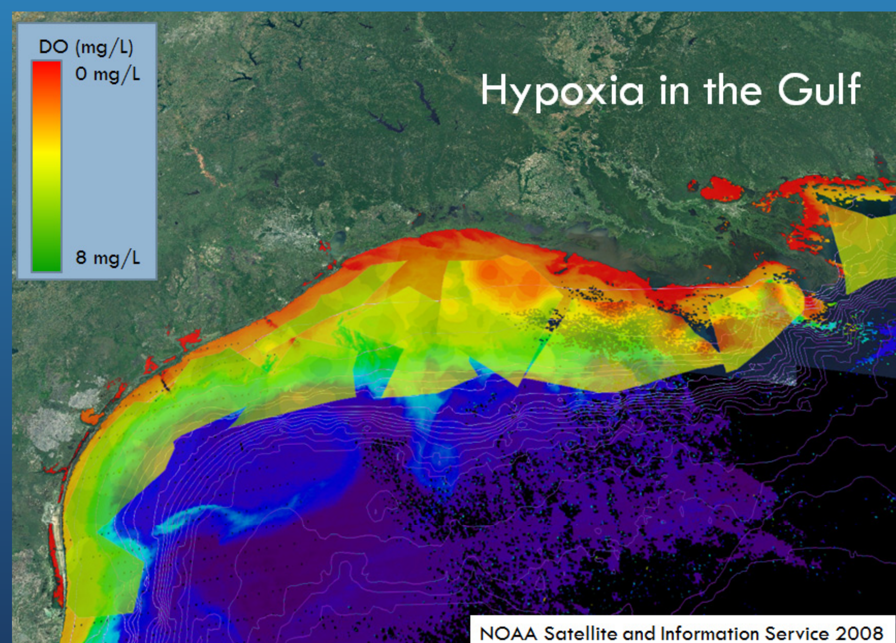
- Why do we want a groundwater transit time distribution for a watershed?
- How large cells in MODFLOW/MODPATH can degrade the distribution and how to fix that. *Abrams, 2012 (under review)*
- How to make sure MODPATH treats weak sinks correctly. *Abrams, Haitjema, and Kauffman, 2012 (under review)*

Motivation: Transit time distributions form the foundation for nitrate response functions.



http://wi.water.usgs.gov/rna/9km30/images/sparrow_rna.gif

Eutrophication

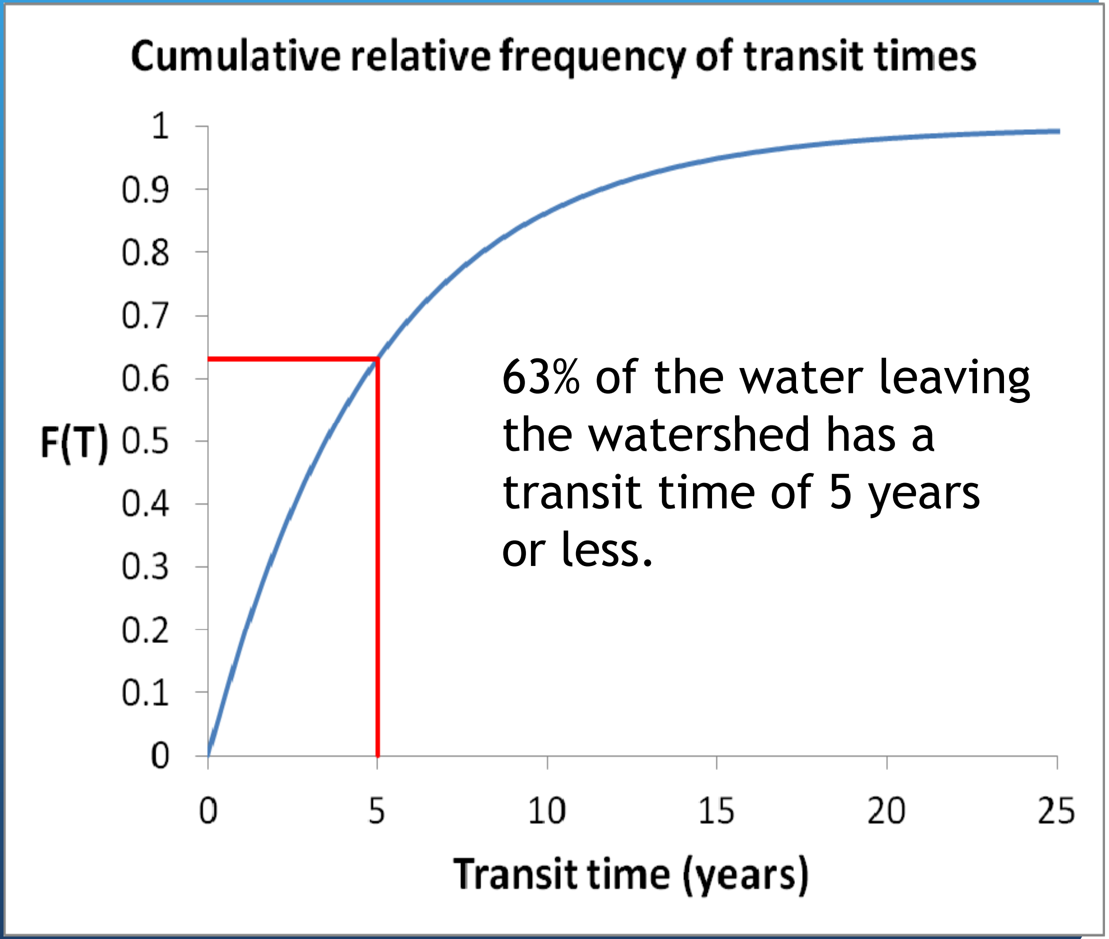


NOAA Satellite and Information Service 2008

Transit time distribution in a watershed

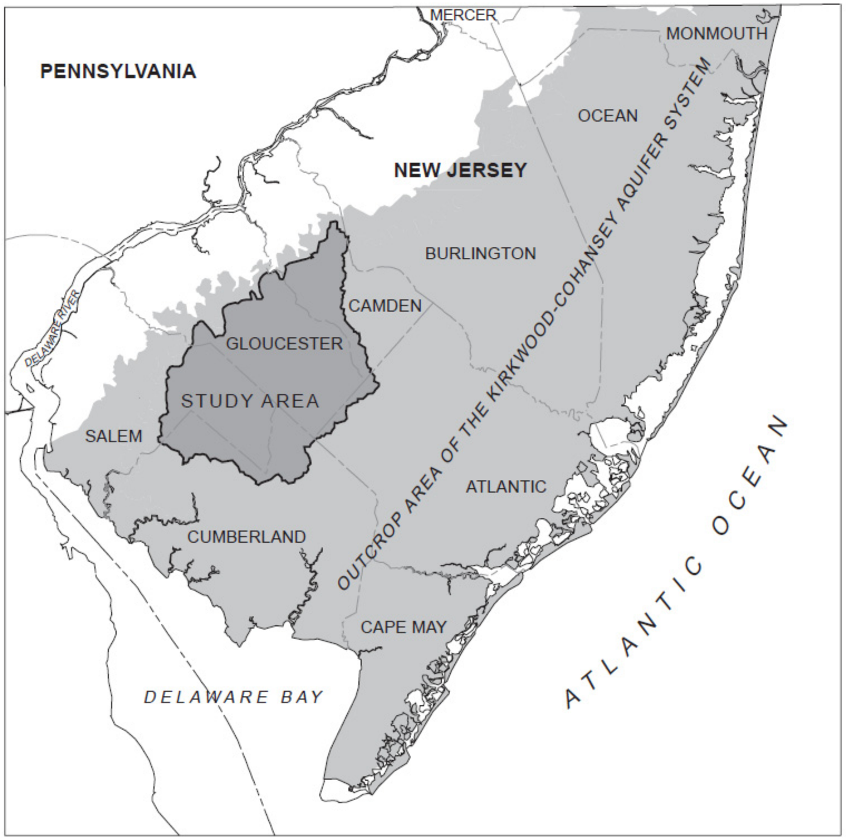
$F(T)=Q(T)/Q_{total}$

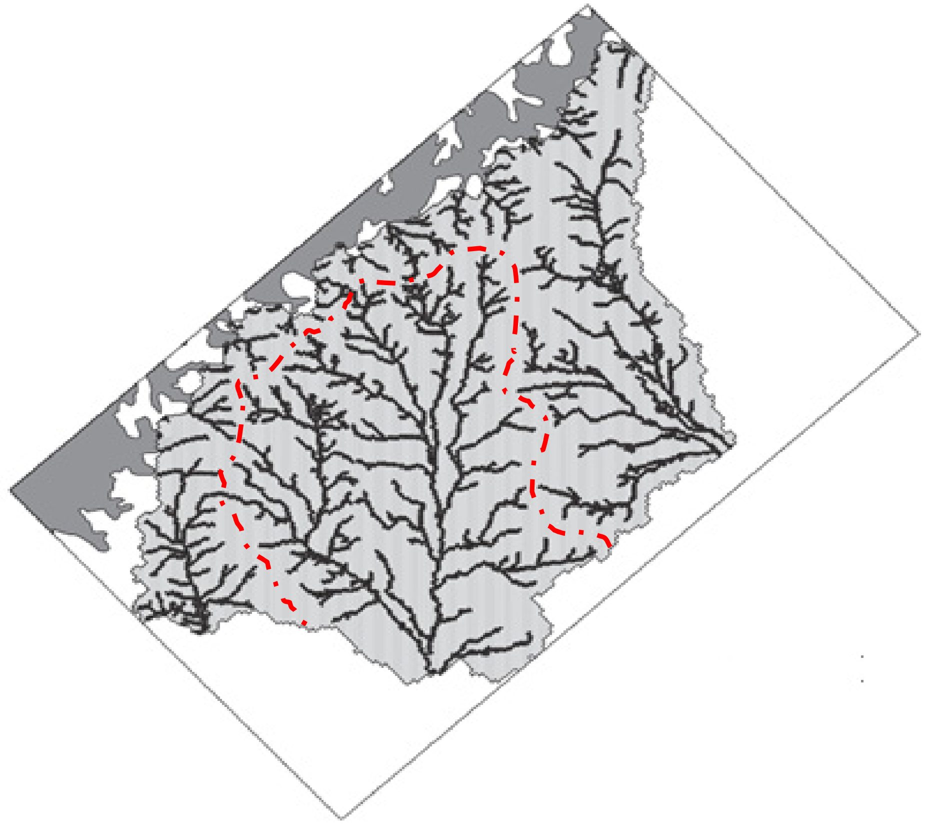
Trace thousands of water particles released uniformly over the watershed. Record transit times to construct the function *F(T)*.



Nitrate concentration in stream effluent $c_e(t)$ is related to the transit time distribution *F'(T) (dF/dT)* by a convolution integral.

$$c_e(t) = \int_0^\infty c(t-T)F'(T)\,dT$$



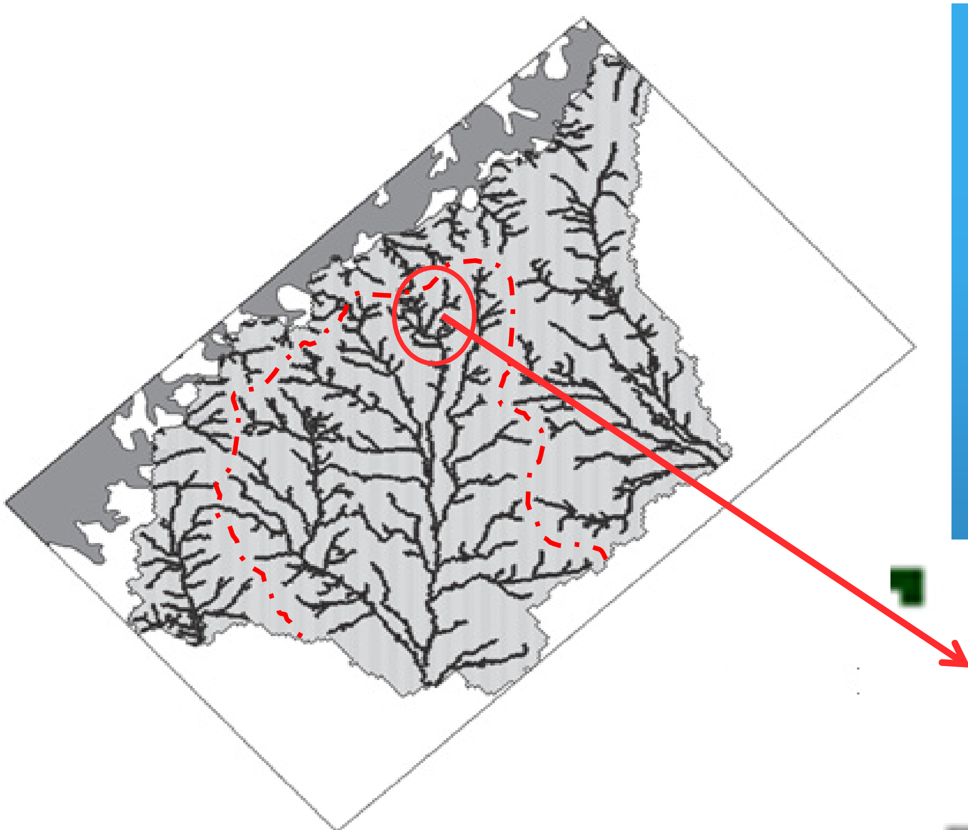


Maurice watershed model by Kauffman et al. 2001 for the National Water Quality Assessment Program (NWQA)

Many regional MODFLOW/MODPATH models lack the grid resolution to accurately represent stream geometry, specifically stream width.

Obvious solution: Make a high resolution model!

But what if you are stuck with a coarse model?



Large river cells are treated correctly in MODFLOW by use of a “leakance factor” that includes the correct stream dimensions. However, these large cells lead to inaccurate transit times in MODPATH.

Close up of model area shows river cells that are too large.

Instead of redoing this model in high resolution, we will try to “fix” the transit time distribution produced by the coarse MODPATH model.

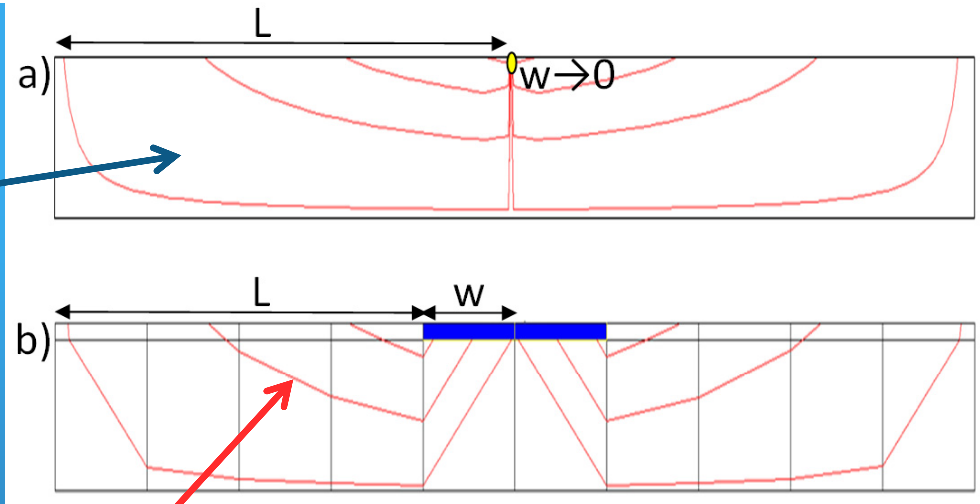
Consider 1D flow case

$$F(T) = 1 - \exp(-T / \bar{T})$$

Eriksson, 1958
Gelhar & Wilson, 1974

$$\bar{T} = \frac{nH}{N}$$

n= porosity, *H*=thickness, *N*=recharge



Flow in a one-dimensional aquifer

$$F^*(T) = 1 - e^{-\frac{T}{(1+a)\bar{T}}}$$

Haitjema, 1995

$a = w/L$

We get *F*(T)* from a coarse model, but we want *F(T)*!

What MODPATH gives us

What we want

$$F^*(T) = 1 - e^{-\frac{T}{(1+a)\bar{T}}} \Rightarrow F(T) = 1 - e^{-\frac{T}{\bar{T}}}$$

Let's introduce a new mean transit time…

$$\bar{T}^* = \frac{\bar{T}}{1+a}$$

So that…

$$F^*(T) = 1 - e^{-\frac{T}{(1+a)\bar{T}^*}} = 1 - e^{-\frac{(1+a)T}{(1+a)\bar{T}}}$$
$$= 1 - e^{-\frac{T}{\bar{T}}} = F(T)$$

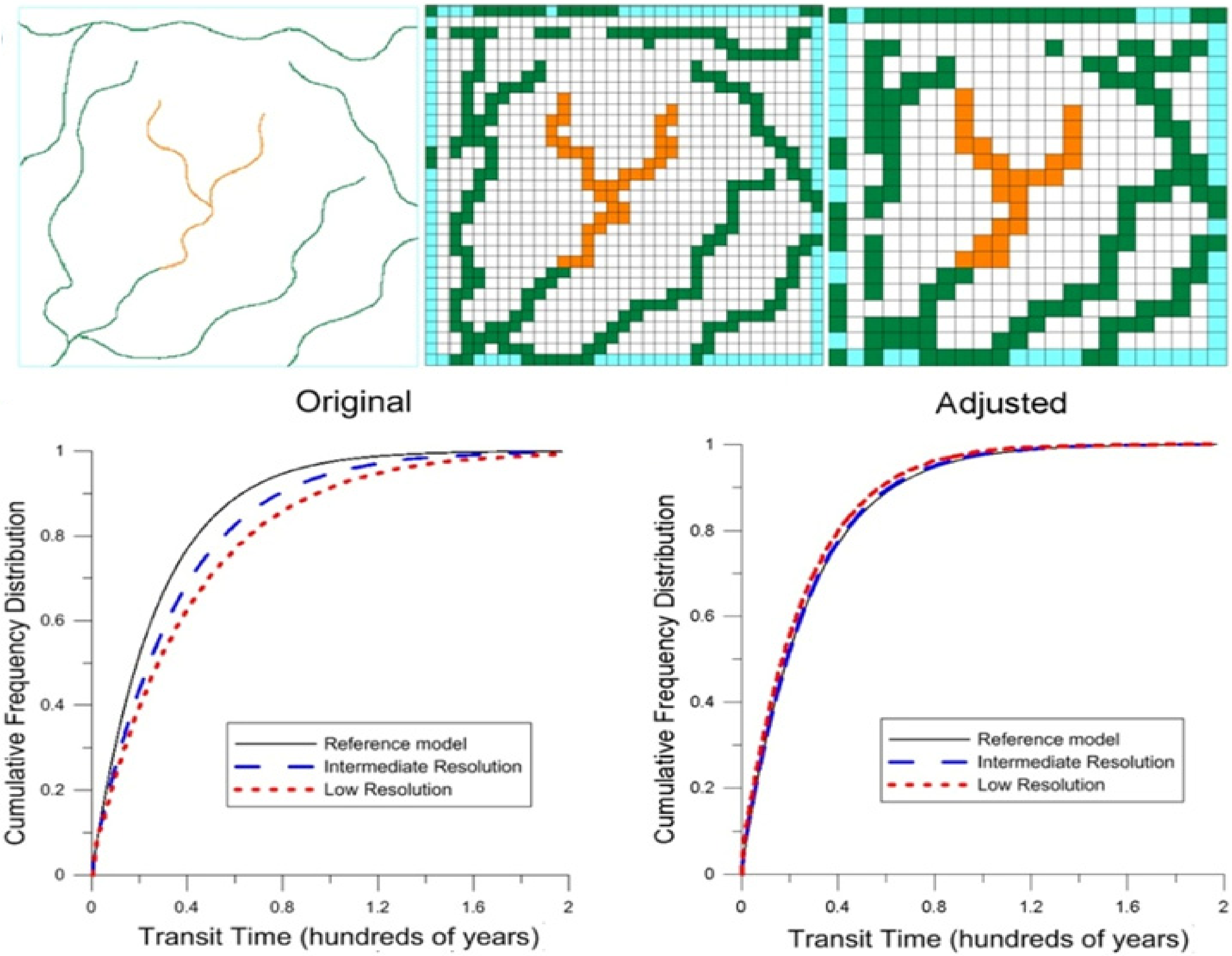
Application to 2D flow

Changing the mean transit time by multiplication of $1/(1+a)$ is achieved in MODPATH by post processing all particle traces. Thus all transit times will be multiplied by $1/(1+a)$.

In a 1D model the factor $1/(1+a)$ is the stream width divided by the total aquifer length.
In a 2D model we *interpret* the factor $1/(1+a)$ as the area of all stream cells divided by the total watershed area.

Since we distribute particles uniformly over the watershed we can also interpret $1/(1+a)$ as the # of particles on stream cells divided by the total number of particles released.

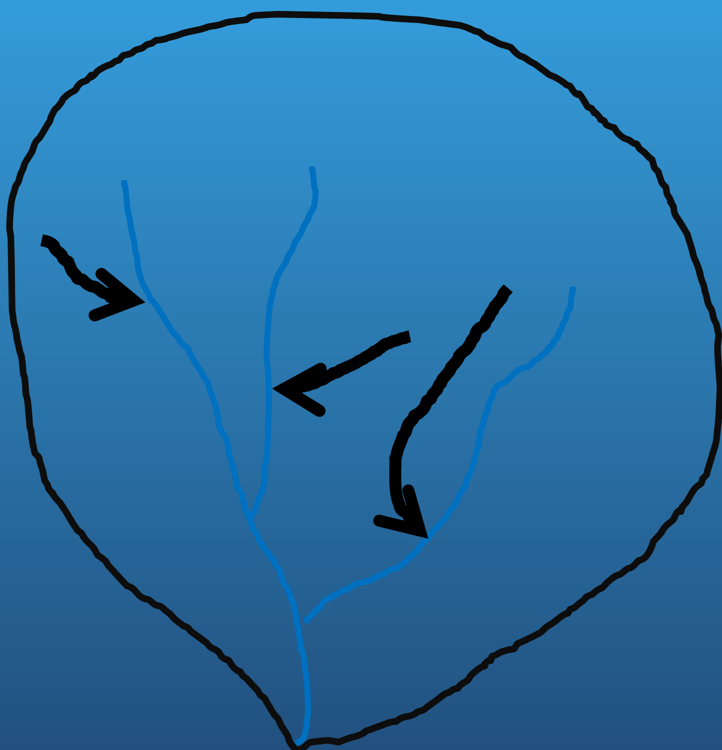
Note:
1) Only for $1/(1+a) < 0.95$ is the correction noticeable.
2) Correction is exact for 1D flow and approximate for 2D flow.


Original
Adjusted
Cumulative Frequency Distribution
Transit Time (hundreds of years)
Reference model
Intermediate Resolution
Low Resolution
0
0.2
0.4
0.6
0.8
1
0
0.4
0.8
1.2
1.6
2
Cumulative Frequency Distribution
Transit Time (hundreds of years)
Reference model
Intermediate Resolution
Low Resolution

What is the story about weak sinks?

Haitjema (1995) showed that the cumulative relative frequency distribution of transit times (CRFD) for an <u>idealized</u> watershed is the same as for 1D flow:

Plan View



$$F(T) = 1 - \exp(-T / \bar{T})$$

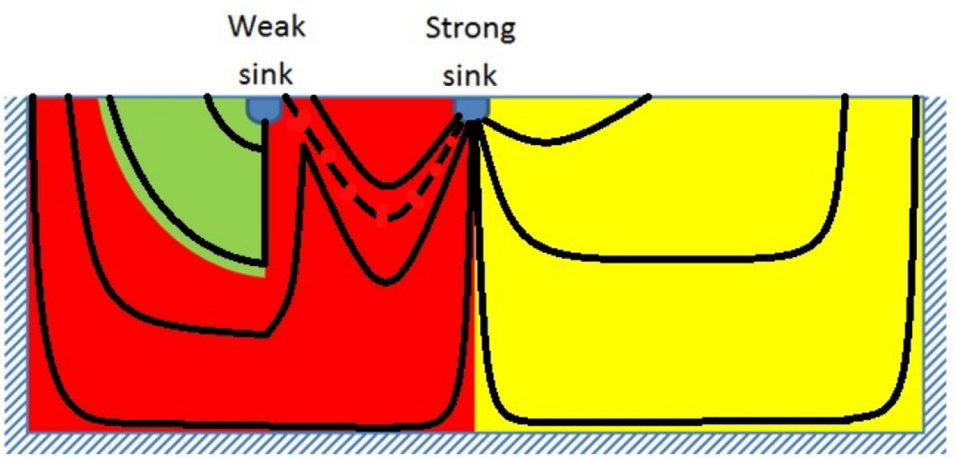
Regardless of watershed size, shape, stream network, and hydraulic conductivity distribution....

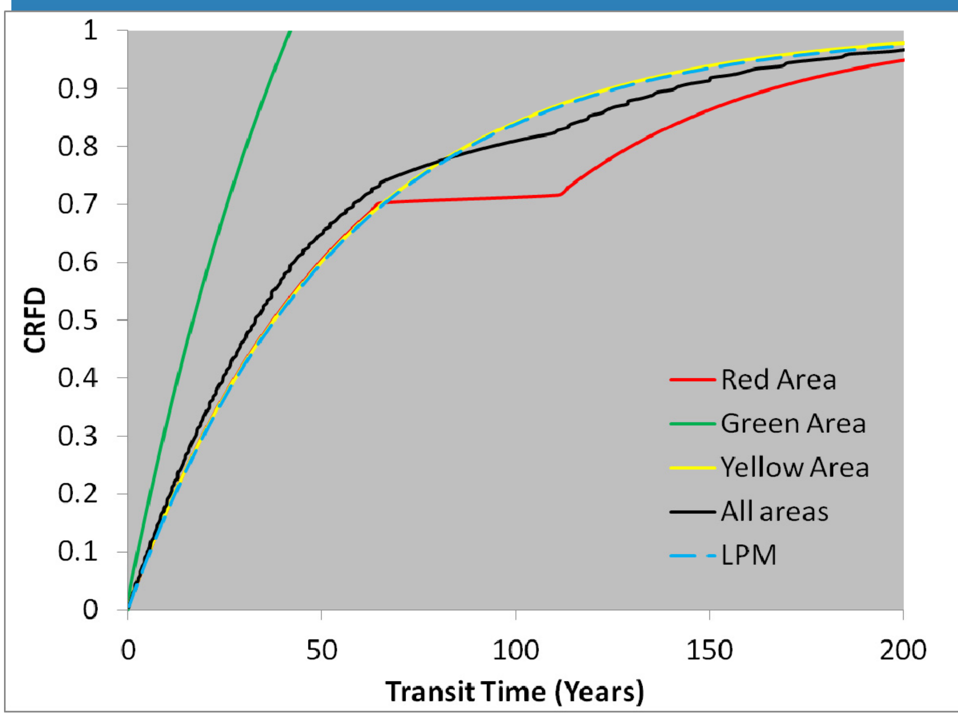
However, *F(T)* does **not** apply in the presence of **weak sinks**!

Weak sinks are streams (wells) which do not extract water over the full aquifer height.

So we must model the CRFD in MODPATH, since it is not *F(T)* !

1D flow into a weak sink and strong sink





The green area represents water discharging to a weak sink.

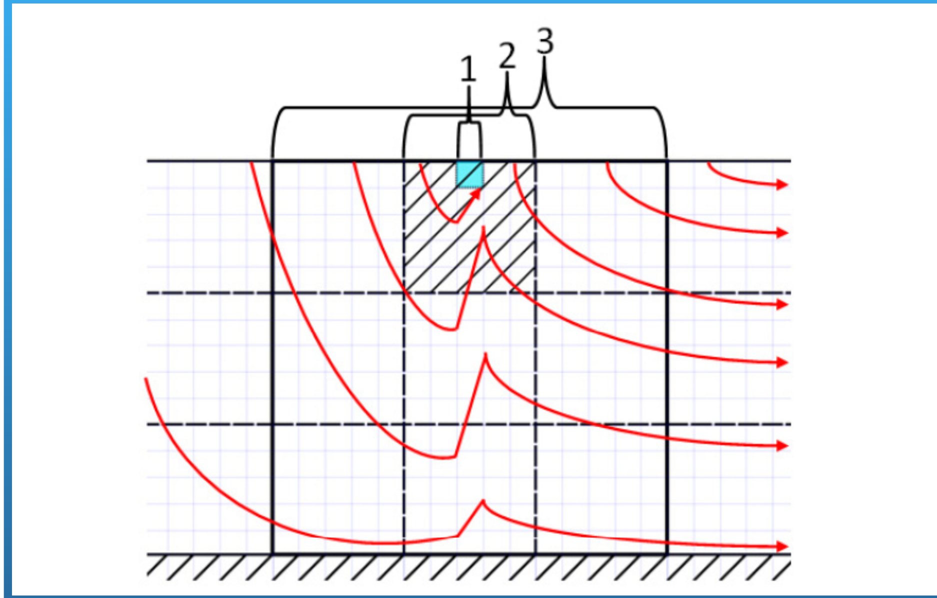
The red area represents water discharging to a strong sink but with ages influenced by a weak sink.

The yellow area represents water discharging to a strong sink with an age distribution that increases monotonously with depth.

The **black** curve represents the combined CRFD for the aquifer.

LPM (lumped parameter model) is the exponential CRFD (Haitjema, 1995).

Distinguish between "weak sinks" and "weak-sink cells".



1) **High resolution model: All weak sink streams are strong sink cells.**

1) **Medium resolution model: Some weak sink streams are weak sink cells others may be strong sink cells.**

1) **Single layer model: All weak sink streams are also weak sink cells.**

A **strong sink cell** discharges all water that enters the cell, hence stops all particles arriving at the cell.

A **weak sink cell** does not discharge all water that enters the cell, so when do we stop a particle and when do we let it pass?

MODPATH settings for weak sinks

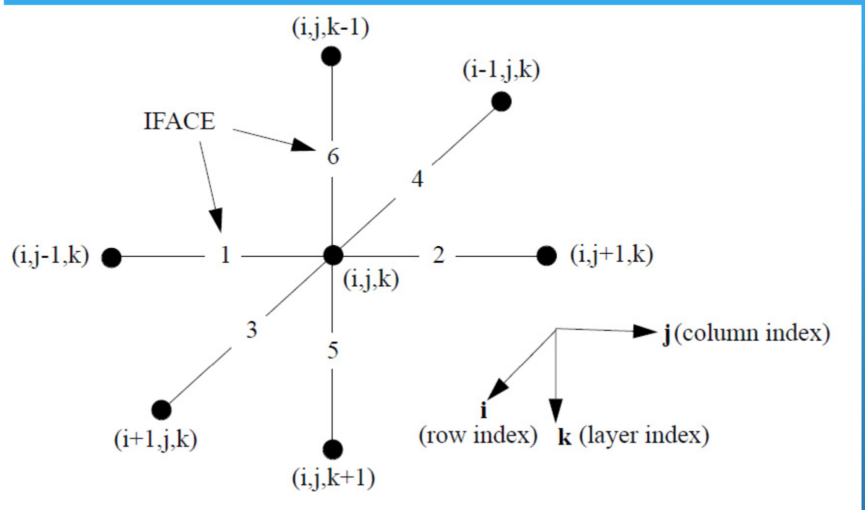


Image from Pollock 1994

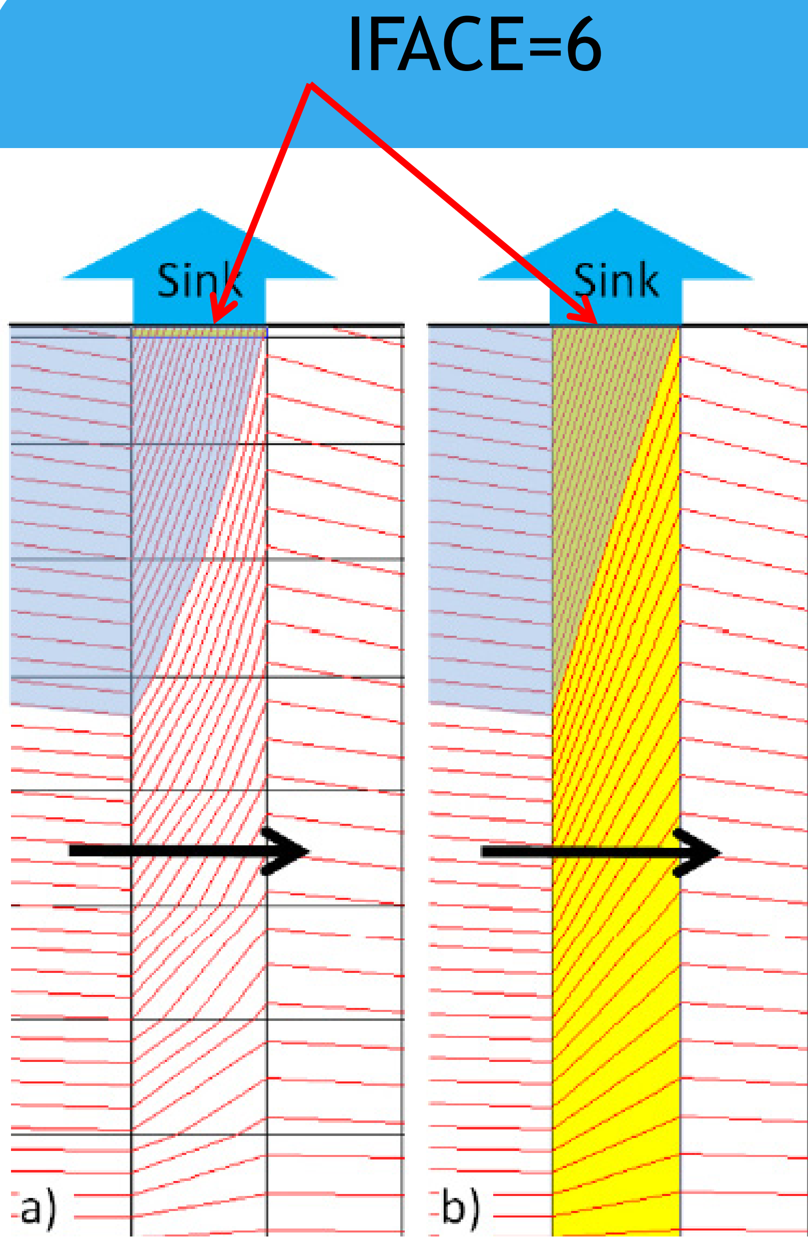
3 possible global settings:

- STOP all particles at a weak sink
- STOP when relative strength ***>S****
- PASS all particles through a weak sink

* e.g. *S=0.5* means 50% of water flowing under the weak sink is discharged.

Particles will be stopped at the cell face defined by the "IFACE" parameter in MODPATH. The default is IFACE=0, which means that the particle will be stopped at the first cell face it reaches.

There is much confusion about how to treat weak sinks in MODPATH. For instance, Visser et al. (2009) modified MODPATH to split particles (Splitpath) at weak sinks with relative weights according to *S*. Particles with weight *S* are stopped and with weight *(1-S)* are passed. This does not work!!



Actual pathline traces in MODPATH

Particle traces in MODPATH are ordered in the vertical cross-section; generally from short transit times high up in the aquifer to longer transit times deeper in the aquifer. This order must be preserved when skimming off water with weak sinks. Splitpath (Visser et al. 2009) does not discriminate between these pathlines, it stops and passes water of all ages arriving at the weak sink.

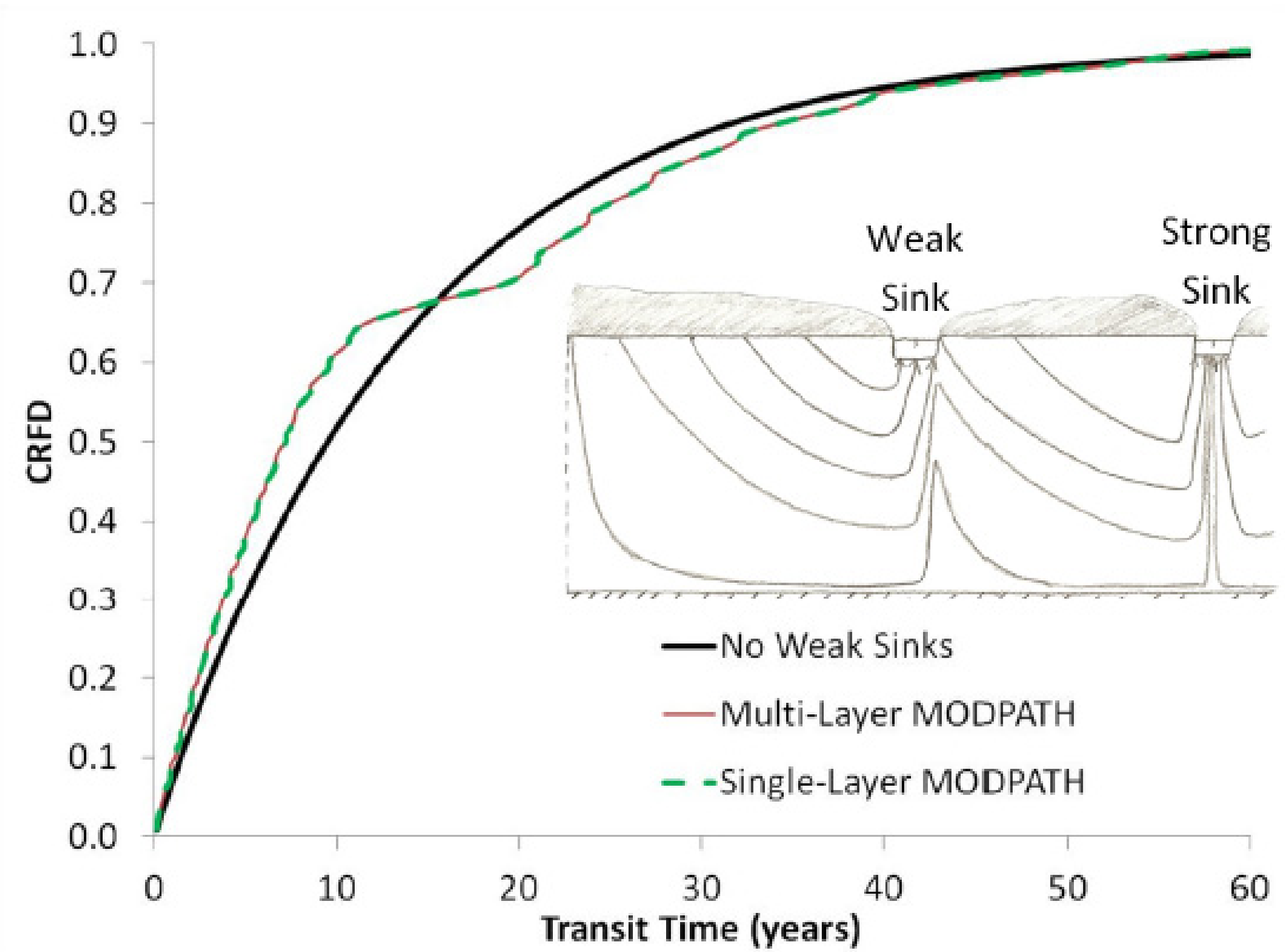
Proper MODPATH settings for weak-sink streams:

- IFACE=6

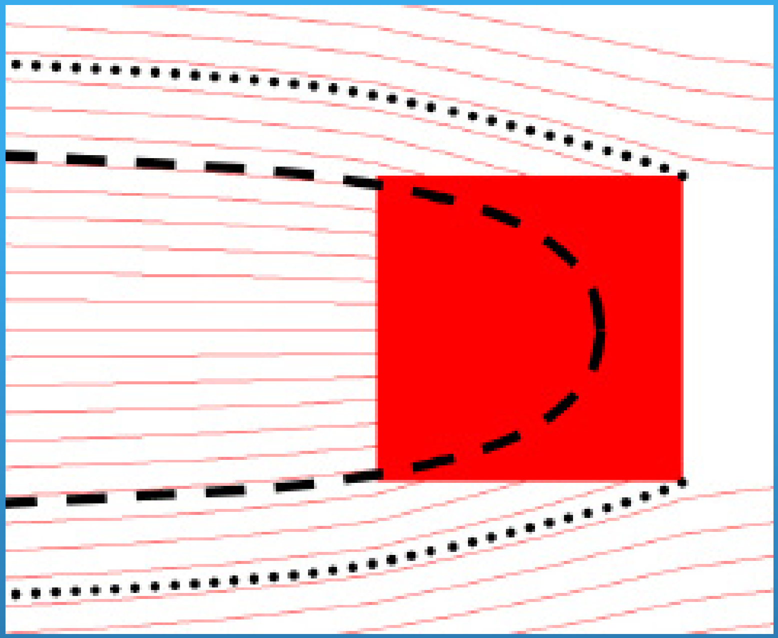
- STOP all particles

Assuming sink cell is in upper layer.

“IFACE=6” and “STOP all particles” yields the correct CRFD, even in a single-layer model.

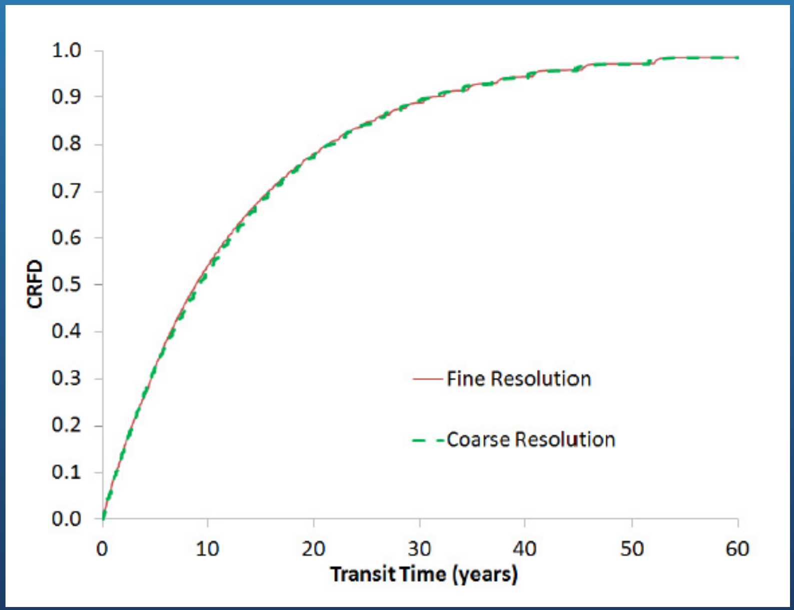


What about wells?



Strong-sink wells often end up in weak-sink cells. Weak-sink well cells will show too large a capture zone in MODPATH.

Several strategies exist to overcome this well cell-size problem. Spitz et al. (2001) use a nested refined grid, while Zheng (1994) uses an analytic solution inside the weak-sink well cell.



Surprisingly, with “IFACE=0” and “STOP all particles” the cumulative *relative* frequency distribution of transit times (CRFD) for the well is still correct – *without any special measures*!

This is assuming that the CRFD in the large capture zone is the same as in the actual capture zone.

Conclusions

- Coarse MODFLOW/MODPATH models can still produce good transit time distributions for a watershed by use of a simple post processing of the particle transit times.
- Weak-sink streams can be modeled accurately by MODPATH provided the settings “IFACE=6” and “STOP all particles” are used.
- Weak-sink well cells in MODPATH generally do not need any corrections to provide the correct transit time distribution provided “IFACE=0” and “STOP all particles” are used.